Absorbing Boundary Conditions for the Modeling of Scatterers in Parallel-Plate Transmission Media

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***Abstract*—In modeling wave propagation using the Finite Difference Time Domain (FDTD) method, the field gradients at each point are calculated using central difference approximation. For points on the edge of the model volume a separate algorithm to simulate absorbing boundaries has to be used. Most of these absorbing boundary conditions assume normal wave incidence at the boundary, but at the sides of stripline or microstrip this is not the case. This letter proposes a variant of dispersive boundary conditions with one phase velocity given an infinite value to represent a wave travelling parallel to the boundary. When applied to uniform stripline, these boundary conditions gave better results than the original version of the dispersive boundary condition with two finite phase velocities. These boundary conditions enable the use of a smaller calculation volume, thus saving computing memory and time.**

I. Introduction

The finite-difference time-domain (FDTD) method first developed by Yee [1] in 1966 provides a flexible and relatively simple way of modeling the propagation of waves in objects of arbitrary geometry, by discretising Maxwell's curl equations in space and time. Once initial values of electric field E and magnetic field H are known, fields at subsequent times are calculated from these values and their gradients. However, if the volume under consideration has an open boundary, a way has to be found of obtaining the tangential E values there, as these cannot be obtained from the central difference equations on which the method relies. This has usually been done by assuming normal incidence [2], [3], or some known characteristic of the field in a similar but simpler geometry [4], [5].

The method proposed here is an adaptation of the dispersive boundary conditions first developed by Bi *et al.* [6], [7] which would be particularly suitable for the modeling of discontinuities and scatterers in otherwise uniform transmission media such as stripline or microstrip.

II. Dispersive Boundary Conditions

Following the method of Mur [2], the values of E_x, E_y and E_z at the boundaries are found by assuming the field components satisfy the wave equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{1}$$

where v is the velocity of the wave. For a plane wave travelling out of the region of interest towards the $x = 0$ plane with inverse velocity components $s_y = v_y/v^2, s_z = v_z/v^2, s_x = [1-(vs_y)^2-(vs_z)^2]^{1/2}$, the first-order boundary condition at $x = 0$ is

$$\frac{\partial E}{\partial x} - \frac{1}{v}\left[1-(vs_y)^2-(vs_z)^2\right]^{1/2}\frac{\partial E}{\partial t} = 0. \tag{2}$$

For normal incidence, $s_x \gg s_y, s_z$, so that the first-order boundary condition for a normally incident wave with a single phase velocity is simply

$$\frac{\partial E}{\partial x} - \frac{1}{v}\frac{\partial E}{\partial t} = 0. \tag{3}$$

This expression can be extended to include more than one phase velocity [6]. For two phase velocities v_1 and v_2

$$\left[\frac{\partial}{\partial x} - \frac{1}{v_1}\frac{\partial}{\partial t}\right]\left[\frac{\partial}{\partial x} - \frac{1}{v_2}\frac{\partial}{\partial t}\right]E = 0. \tag{4}$$

If v_1 represents a wave travelling normally toward $x = 0$, while v_2 represents a wave travelling in the z-direction, i.e. parallel to $x = 0$, then v_2 is effectively infinite and we can write

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{v_1}\frac{\partial^2 E}{\partial x \partial t} = 0. \tag{5}$$

Discretizing this expression for values of E and its gradients at a point a distance dx from the $x = 0$ plane at time $t = p\, dt$ and re-arranging gives

$$\begin{aligned} E^{p+1}(0) = {} & 2E^p(1) - E^{p-1}(2) - (1+\gamma) \\ & \cdot [E^p(0) - E^{p+1}(1) - E^{p-1}(1) + E^p(2)] \\ & - \gamma[E^{p+1}(0) - 2E^p(1) + E^{p+1}(2)] \end{aligned} \tag{6}$$

where $\gamma = \delta x - v_1\delta t/\delta x + v_1\delta t$.

III. Results

Two sets of dispersive boundary conditions were applied as BC1 in Fig. 1 to an FDTD simulation of a stripline. The Stripline dimensions are: width $w = 28$ mm, ground plate spacing $b = 12.7$ mm, and ground plate width $g = 37.5$ mm, leaving gaps of just 4.75 mm between the strip edges and the boundary. This example is an extreme case, with the field strength still quite substantial at the boundaries. For comparison, a strip with a wide volume, leaving a gap of 36 mm between the strip edge and the absorbing boundary, was also simulated.

Fig. 2 shows the cross-sectional field pattern obtained for $v_1 = c, v_2 = \infty$, and for $v_1 = c, v_2 = 2.8 \times 10^8$ m/s. In the second case the boundary conditions have reduced the field strength near the edges. For each case, two simulations were run: one starting with an approximation to the field pattern with a Gaussian z-dependence which was run to adjust itself to the true field pattern, and the second using this 'self-filtered' field

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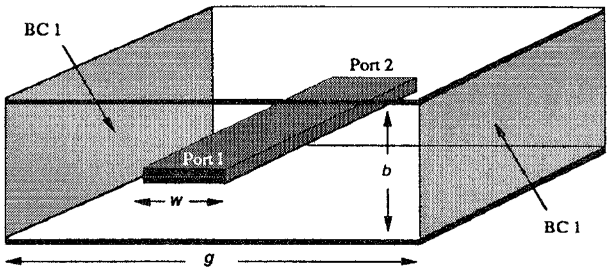

Fig. 1. Stripline structure modelled.

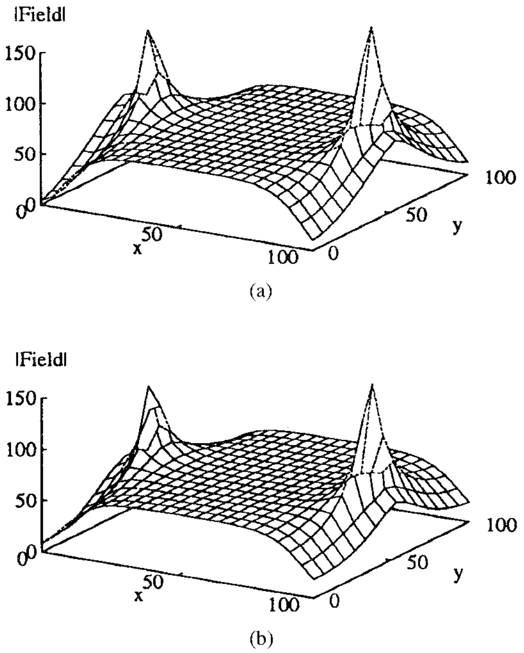

Fig. 2. Cross-sectional electric field in stripline modelled using dispersive boundary conditions with (a) $v_2 = \infty$, and (b) two finite velocities.

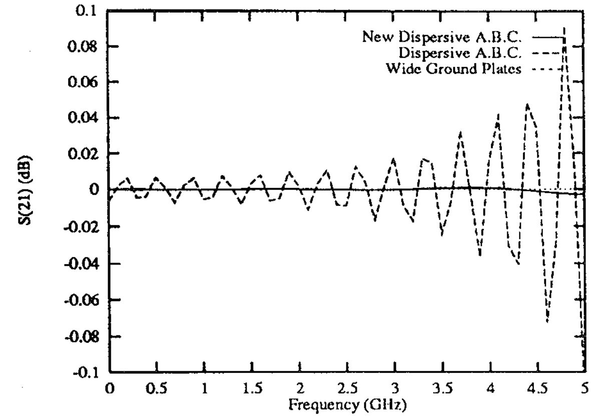

Fig. 3. Transmission coefficient S_{21} versus frequency for uniform striplines modelled with conventional dispersive boundary conditions (- - -), new dispersive boundary conditions with $v_2 = \infty$ (_____), and with wide modelled volume (...).

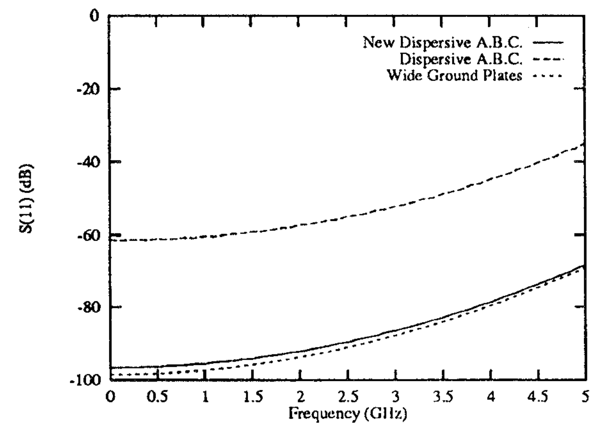

Fig. 4. Reflection coefficient S_{11} versus frequency for uniform striplines modelled with conventional dispersive boundary conditions (- - -), and new dispersive boundary conditions with $v_2 = \infty$ (_____).

pattern was run in a uniform stripline and monitored 1500 and 2000 mm along the line.

From these time-series signals the S-parameters were obtained by Fourier Transform. Ideally $S_{21} = 0$ dB and $S_{11} = -\infty$ dB, and Figs. 3 and 4 show that for wide ground-plates this is the case, within the numerical limits of the FDTD simulation. For the narrow stripline, the dispersive boundary condition with $v_2 = \infty$ gives results similar to those for the wide volume, showing that the error due to the truncation of the volume is negligible.

IV. Conclusion

This letter has shown that dispersive boundary conditions with one phase velocity set to infinity are particularly suitable for the modeling of stripline and microstrip. The infinite phase velocity component prevents the boundary conditions from artificially absorbing energy from waves running parallel to the boundaries, i.e. the stripline TEM mode. The finite velocity component allows for the absorbtion of any scattered energy, as might arise from discontinuities. The use of these boundary conditions enables a reduction of the width of the volume that it is necessary to model, thus saving memory and computing time. The Superabsorbtion method [8] can be applied to these boundary conditions to improve their accuracy still further.

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